H∞ Control of a Bidirectional Converter Based on Novel Interval Type-2 T-S Fuzzy Model*

Gwo-Ruey Yu and Z.-Y. Chen

Abstract—**This paper proposes novel interval type-2 (IT-2) T-S fuzzy control systems and designs H∞ controllers for a DC/DC converter. The novel IT-2 T-S fuzzy control systems proposed in this paper can divide the nonlinear converter into two subsystems. Therefore, regardless of the number of premise parameters the state-space equations have, the novel IT-2 T-S fuzzy control systems only need two control rules. It does not only reduce the number of the control rules, but it also reduces the calculation time of the microprocessor and has more relaxed stability conditions. The H∞ performance index is used to suppress the external interference and the forward diode bias of buck mode and boost mode. The experimental results show that the novel IT-2 T-S fuzzy control systems have better performance than the existing IT-2 T-S fuzzy control systems.**

I. INTRODUCTION

Takagi and Sugeno proposed the T-S fuzzy control methodology, which approximates the nonlinear plant through multiple sets of linear subsystems and fuzzy rules. This approach formulates fuzzy rules through IF-THEN, divides the nonlinear system into several subsystems, de-fuzzifies the controller output through parallel distributed compensation. The stability conditions of T-S fuzzy control system are expressed in the form of linear matrix inequality (LMI) for finding control gains. The T-S fuzzy control has been applied to a two-way charger and discharger, and the stability of the system is guaranteed through the Lyapunov theorem [1].

However, as the T-1 fuzzy logic system did not consider the uncertainty of the system model, Mendel proposed the IT-2 fuzzy logic system, using the upper and lower bound membership function to cover the model uncertainty, making the controlled system closer to the real situation. The number of rules of the IT-2 T-S fuzzy systems will increase with the number of antecedent variables. If the number of rules is too large, the size of the solution space of LMI stability conditions will be limited, making it difficult to find solutions with better performance. Furthermore, as the number of rules increases, it also leads to an increase in the time of chip computing.

Therefore, this paper proposes the novel IT-2 T-S fuzzy control systems. The existing establishment approach of IT-2 T-S fuzzy model rule was improved. By considering the maximum and minimum values of the antecedent variable, the number of rules of the existing IT-2 T-S fuzzy model was reduced from 2α to 2 rules, where α is the number of the antecedent variable. The novel IT-2 T-S fuzzy control systems can divide the nonlinear bidirectional converter into two subsystems. The control rules can be greatly reduced to two, which loosens the LMI-based stability conditions. Hence, a larger feasible solution space is obtained to promise favored control gains can be explored with greater performance. Since the forward bias of the diode in the bidirectional converter circuit could be regarded as a kind of fixed interference signal, the H∞ performance index is added to the proposed stability theorem so that the bidirectional converter has the ability to suppress external interference and the forward diode bias in either buck mode or boost mode.

II. NOVEL IT-2 T-S FUZZY H∞ CONTROL

A. Novel IT-2 T-S Fuzzy Model of a Bidirectional Converter

Fig. 1 shows the circuit of the bidirectional converter

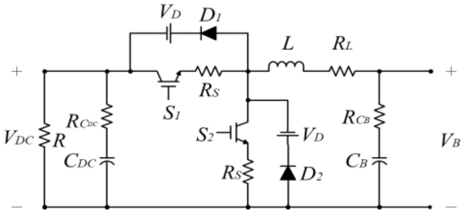

Figure 1. Non-ideal DC-DC bidirectional converter.

Using IF-THEN fuzzy rules, the novel IT-2 T-S fuzzy model considers the membership function in each column as a coefficient so as to form a diagonal matrix and divides the dynamic system model into two subsystems as follows:

Model Rule 1: IF *z₁* is *m₁₁*(*z*(*t*)) and *z₂* is *m₂₁*(*z*(*t*)), THEN

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_{C_{DC}} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} m_{21} & 0 & 0 \\ 0 & m_{11} & 0 \\ 0 & 0 & m_{11} \end{bmatrix} \left\{ \begin{bmatrix} -\frac{1}{L}\left(R_L + \frac{RR_{C_{DC}}}{R+R_{CDC}}\right) & -\frac{1}{L}\left(\frac{R}{R+R_{C_{DC}}}\right) & 0 \\ \frac{1}{C_{DC}}\left(\frac{R}{R+R_{C_{DC}}}\right) & -\frac{1}{C_{DC}}\left(\frac{1}{R+R_{C_{DC}}}\right) & 0 \\ -\frac{RR_{C_{DC}}}{R+R_{C_{DC}}} & -\frac{R}{R+R_{C_{DC}}} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_{C_{DC}} \\ x_{err} \end{bmatrix} + \begin{bmatrix} \frac{1}{L}\left[N_{2\max} + V_D\right] \\ -\frac{1}{C_{DC}}\left(\frac{1}{R+RR_{C_{DC}}}\right) N_{1\max} \\ \frac{RR_{C_{DC}}}{R+R_{C_{DC}}} N_{1\max} \end{bmatrix} D + \begin{bmatrix} \frac{V_B - V_D}{40L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{r}{100} \end{bmatrix} \begin{bmatrix} 40 \\ 0 \\ 100 \end{bmatrix} \right\} \tag{1}$$

Model Rule 2: IF *z₁* is *m₁₂*(*z*(*t*)) and *z₂* is *m₂₂*(*z*(*t*)), THEN

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_{C_{DC}} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} m_{22} & 0 & 0 \\ 0 & m_{12} & 0 \\ 0 & 0 & m_{12} \end{bmatrix} \left\{ \begin{bmatrix} -\frac{1}{L}\left(R_L + \frac{RR_{C_{DC}}}{R+R_{CDC}}\right) & -\frac{1}{L}\left(\frac{R}{R+R_{C_{DC}}}\right) & 0 \\ \frac{1}{C_{DC}}\left(\frac{R}{R+R_{C_{DC}}}\right) & -\frac{1}{C_{DC}}\left(\frac{1}{R+R_{C_{DC}}}\right) & 0 \\ -\frac{RR_{C_{DC}}}{R+R_{C_{DC}}} & -\frac{R}{R+R_{C_{DC}}} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_{C_{DC}} \\ x_{err} \end{bmatrix} + \begin{bmatrix} \frac{1}{L}\left[N_{2\min} + V_D\right] \\ -\frac{1}{C_{DC}}\left(\frac{1}{R+RR_{C_{DC}}}\right) N_{1\min} \\ \frac{RR_{C_{DC}}}{R+R_{C_{DC}}} N_{1\min} \end{bmatrix} D + \begin{bmatrix} \frac{V_B - V_D}{40L} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{r}{100} \end{bmatrix} \begin{bmatrix} 40 \\ 0 \\ 100 \end{bmatrix} \right\} \tag{2}$$

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B. *Novel H∞ LMI-based Stability Conditions*

The H∞ performance index denotes the system output $y(t)$ is subject to disturbance effects $v(t)$ attenuated to γ level.

$$\sup_{\|v(t)\|_2\neq 0} \|y(t)\|_2 / \|v(t)\|_2 \leq \gamma,\ 0 \leq \gamma \leq 1 \tag{3}$$

Theorem: The bidirectional converter control system will be stable with H∞ performance if there exists a positive definite constant matrix P, matrix M_j, such that the following LMI conditions are satisfied.

$$\begin{bmatrix} -\phi_{iiii} & * & * \\ -\hat{E}_i^T & \gamma^2 I & 0 \\ (mC_i + nC_j)X & 0 & I \end{bmatrix} > 0, \quad \forall i = j = k = l \tag{4}$$

$$\begin{bmatrix} -(\phi_{ijjj} + \phi_{jiii}) & * & * \\ -2\hat{E}_i^T & 2\gamma^2 I & 0 \\ (mC_i + nC_j)X & 0 & 2I \end{bmatrix} > 0, \quad \forall i < j = k = l \tag{5}$$

$$\begin{bmatrix} -\left(\phi_{ijkk} + \phi_{ikjj}\right) & * & * \\ -2\hat{E}_i^T & 2\gamma^2 I & 0 \\ (2mC_i + n(C_i + C_j))X & 0 & 2I \end{bmatrix} > 0, \quad \forall j < k = l \tag{6}$$

$$\begin{bmatrix} -(\phi_{ijkl} + \phi_{ijlk}) & * & * \\ -2\hat{E}_i^T & 2\gamma^2 I & 0 \\ 2(mC_i + nC_j)X & 0 & 2I \end{bmatrix} > 0, \quad \forall k < l \tag{7}$$

III. RESULTS

A. *Comparison of LMI Solution Space*

This section compares the solution space of the novel IT-2 T-S fuzzy control systems with the existing IT-2 T-S fuzzy control systems, as shown in Fig. 2. Table I lists the number of fuzzy rules and the number of LMI inequalities. The novel IT-2 T-S fuzzy control systems solution space is larger than that of the existing IT-2 T-S fuzzy control systems because the proposed approach greatly reduces the number of rules and LMI-based stability conditions. The stability conditions should be more relaxed, and the solution space should be made larger to find more excellent solutions.

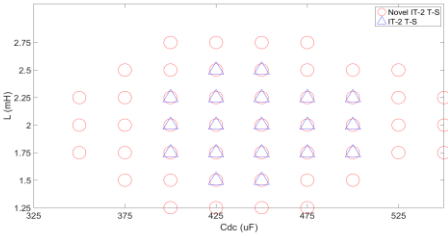


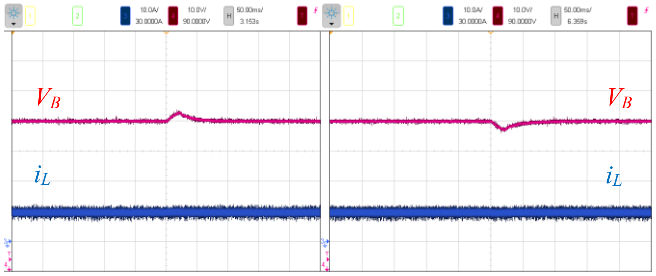
Figure 2. Comparison of LMI solution spaces.

TABLE I. NUMBER OF FUZZY RULES AND LMIS

Control systems	Number of Rules	Number of LMIs
Novel IT-2 T-S	2	10
Existing IT-2 T-S	4	130

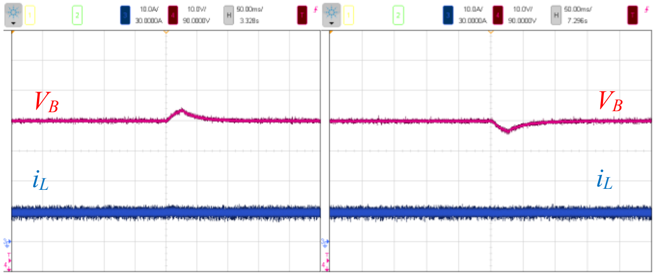
B. *Experiments of Input Voltage Variation*

The operating environment is that the input voltage is switched from 370V to 390V and then back to 370V. The responses of the novel IT-2 T-S fuzzy control systems to the input voltage variation are shown in Fig. 3, and the responses of existing IT-2 T-S fuzzy control systems are shown in Fig. 4. The proposed novel fuzzy control systems own better performance on the input voltage variation than the existing fuzzy control systems, not only the overshoot voltage is smaller, and the settling time is also shorter.



(a) 370V → 390V (b) 390V → 370V

Figure 3. Novel IT-2 T-S fuzzy control systems.

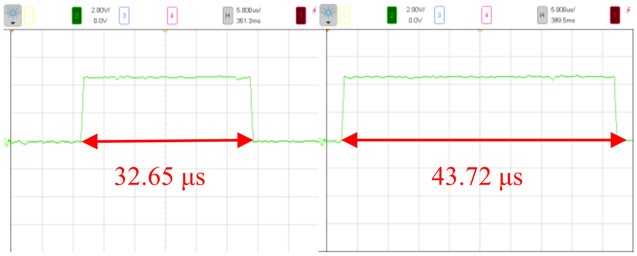


(a) 370V → 390V (b) 390V → 370V

Figure 4. Existing IT-2 T-S fuzzy control systems.

C. *Chip Computational Time*

In this experiment, the general purpose input/output (GPIO) pins of the microprocessor are set as follows. The output is set to be high at the beginning of the algorithm and low at the end of the algorithm so that the computational time of the chip can be measured. Fig. 5 shows a comparison of the experimental results for the proposed and existing approach.



(a) Novel IT-2 T-S fuzzy control (b) Existing IT-2 T-S fuzzy control

Figure 5. Comparison of the computational time.

IV. CONCLUSION

This paper presented the novel IT-2 T-S fuzzy control systems for the bidirectional converter. The novel IT-2 T-S fuzzy controller was designed in consideration of the H∞ performance index. The novel IT-2 T-S fuzzy control systems reduces the number of control rules such that the chip calculation time was decreased and the solution space was relaxed, which makes it easier to determine better performance solutions. The experimental results have validated the effectiveness of the proposed approach.

REFERENCES

[1] G.-R. Yu and H.-R. Kang, "TS fuzzy control of a bidirectional converter," in *Proc. IEEE ICNSC*, 2015, pp. 293-297.