Autoencoder Learning and Variational Gaussian Inference for Predicting Mean Arterial Pressure in Fluid Resuscitation

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***Abstract*— This paper introduces a novel method, called robust nonlinear state space modeling (RNSSM), for predicting hemodynamic responses in fluid resuscitation. The RNSSM approach integrates autoencoder learning and Gaussian inference in a unified framework to address the challenges associated with identifying reliable models with limited and noisy critical care data. Simulation results demonstrate the initial feasibility and performance evidence of the RNSSM approach, which serves as a digital twin of an animal study, in fluid resuscitation scenarios.**

I. INTRODUCTION

Fluid resuscitation is a medical intervention for stabilizing critically ill patients in hypovolemic scenarios. Current fluid management protocols are ad hoc strategies that lack sufficient accuracy to adequately adjust fluid dosing in different clinical scenarios, leading to increased risk of adverse effects [1-3]. While machine learning algorithms have been leveraged for dose-response modeling, they mostly rely on population-based data, limiting their applicability to individual subjects [3]. In our previous study, we designed an individual-based fluid dosing algorithm using model-free reinforcement learning (RL). The model-free RL control approach provided promising results, but it required substantial data for training and showed inferior performance in the presence of clinical disturbances [4].

This paper presents a novel method, called robust nonlinear state space modeling (RNSSM), for predicting hemodynamic responses in fluid resuscitation. It integrates the autoencoder learning and variational Gaussian inference (VGI) in a unified framework to develop subject-specific models for limited, noisy critical care data.

II. METHODOLOGY

We combine the autoencoder learning and VGI to predict individualized mean arterial pressure (MAP) responses to fluid infusion in hemorrhage scenarios. Consider a multiple-input-multiple-output nonlinear state space model in a general form:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= f\left(\boldsymbol{x}_k,\ \boldsymbol{u}_k,\ \boldsymbol{\theta}\right)+\boldsymbol{v}_k \\ \boldsymbol{y}_k &= g\left(\boldsymbol{x}_k,\ \boldsymbol{u}_k,\ \boldsymbol{\theta}\right)+\boldsymbol{\omega}_k \end{aligned} \tag{1}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ represents the hidden state variable, $\boldsymbol{u}_k \in \mathbb{R}^{n_u}$ denotes the observed input, and $\boldsymbol{y}_k \in \mathbb{R}^{n_y}$ represents the measured output. The functions $f(.)$ and $g(.)$ capture the state transition and output measurement, respectively, and $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ represents a vector of unknown parameters. The terms $\boldsymbol{v}_k$ and $\boldsymbol{\omega}_k$ account for process and measurement noise, respectively.

We utilized autoencoders, a type of artificial neural network (ANN), for learning state space models. The autoencoder learns a compressed representation of the input data and reconstructs data back to their original form. We adopt a specific autoencoder structure that learns a nonlinear state space representation of the subject-specific input-output data [5]. Suppose a dataset of input-output $Z=\{u_1,u_2,\dots,u_N,y_1,y_2,\dots,y_N\}$, where u_k is the vector of inputs (fluid dosages) and y_k is the vector of measured outputs (MAP responses). The objective is to find optimal values for functions $e:\mathbb{R}^{n_I}\rightarrow\mathbb{R}^{n_x}$, $f:\mathbb{R}^{n_u}\times\mathbb{R}^{n_x}\rightarrow\mathbb{R}^{n_x}$, and $g:\mathbb{R}^{n_x}\rightarrow\mathbb{R}^{n_y}$ by minimizing the following fitting criterion:

$$\begin{aligned} \min_{e,f,g}\mathcal{L}\left(e,f,\ g,\ Z\right) = \min_{e,f,g}&\sum_{k=k_0}^{N} L\left(\hat{y}_k, y_k\right) \\ \text{s.t.}\quad x_{k+1} &= f\left(x_k,u_k\right) \\ \hat{y}_k &= g\left(x_k\right) \\ x_k &= e\left(I_{k-1}\right) \end{aligned} \tag{2}$$

where e, f, g, are the functions describing the encoder, bridge, and decoder models, Z is the training dataset, $L:\mathbb{R}^{2n_y}\rightarrow\mathbb{R}$ is the loss function, y is the measured input and $\hat{y}_k$ is the predicted output. The information vector $I_k=\left[y_k,\dots,y_{k-n_a+1},u_k,\dots,u_{k-n_b+1}\right]$ represents a subset of the training dataset used to train the encoder. It consists of previous outputs and inputs from a specific time step, k, up to a certain number of previous steps (n_a for outputs and n_b for inputs).

To obtain an acceptable mismatch between the predicted value, $\hat{y}_k$, and the measured value, y_k, we need to design a suitable ANN architecture for training these functions. The autoencoder model, used in this work, consists of three main components: (1) A multilayer ANN encoder for predicting x_k from I_k; (2) A multilayer ANN decoder for predicting y_k from x_k; and (3) A bridge network, also a multilayer ANN model, for modeling the function f that maps x_k to

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x_{k+1}. The state x_{k+1} is constructed by a second autoencoder that maps I_k to x_{k+1} and y_{k+1}.

By incorporating VGI into the autoencoder learning, our goal is to effectively capture uncertainties within the model. In this methodology, the latent vector, the lower dimensional layer of the autoencoder, is represented by the mean and standard deviation of each latent variable. Also, a similarity loss is added to the primary loss function of the network, ensuring that the latent space adheres to a Gaussian distribution. Parameters of the variational distribution are estimated using the Kullback-Leibler (KL) divergence [6]:

$$\begin{aligned} \mathrm{KL} &= D_{KL}(N(\mu_x,\ \sigma_x), N(0, I)) \\ &= -\frac{1}{2}\ \Sigma_{N=1}^{D}\left(1+\log\sigma_x^2-\mu_x^2-\sigma_x^2\right) \end{aligned} \tag{3}$$

where μ_x and σ_x are the mean and standard deviation of the latent vector. The total loss function is defined as:

$$\begin{aligned} \text{Total Loss} &= L_1\left(\hat{O}_k, O_k\right)+L_1\left(\hat{O}_{k+1},\ O_{k+1}\right) \\ &+L_1\left(O_{k+1}^*,\ O_{k+1}\right)+L_2\left(x_{k+1}, \hat{x}_{k+1}\right) \end{aligned} \tag{4}$$

where O_k is the output data, $\hat{O}_k$ is the predicted output of the first decoder, $\hat{O}_{k+1}$ is the predicted output of the second decoder, and O_{k+1}^* is the output of the second decoder when is fed by x_{k+1}^*, the predicted output of the bridge network. Also, $L_2 = \|x, \hat{x}\|_2$ and L_1 is defined as:

$$L_1 = \|x, \hat{x}\|_2 - \frac{1}{2}\Sigma_{N=1}^{D}\left(1+\log\sigma_x^2-\mu_x^2-\sigma_x^2\right) \tag{5}$$

The schematic of the model is illustrated in Fig. 1.

III. Results

The training dataset for RNSSM models was obtained from an animal study conducted at the Resuscitation Research Laboratory, University of Texas Medical Branch [3]. The study involved different sheep undergoing high and medium hemorrhage scenarios along with fluid infusion. The MAP measurements were recorded every 5 minutes, the input data was fluid infusion, and the output data was MAP. The RNSSM model comprised three modules: encoder, decoder, and bridge, each with two hidden layers of 30 neurons.

Fig. 2 shows the predicted MAP against the measured MAP for a sample subject, demonstrating the capability of RNSSM models to predict MAP responses in fluid resuscitation. Table I represents the performance metrics, including root mean square error (RMSE), mean absolute error (MAE), and median absolute percentage error (MDAPE), evaluated for all subjects. Results of Fig. 2 and Table I indicate that the RNSSM model is able to effectively capture the trend and fluctuations of MAP response during hemorrhage resuscitation and encourage further investigation and refinement of the RNSSM framework. The proposed method can potentially tackle the issues caused by the noise and external disturbances, as well as the limited data availability.

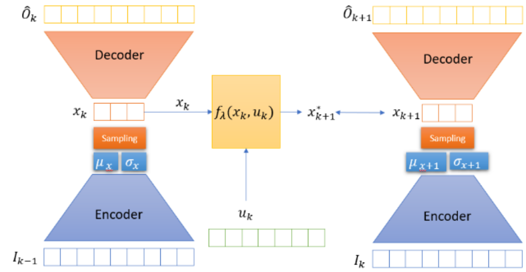

Figure 1. Robust nonlinear state space modeling (RNSSM) approach integrating autoencoder learning and variational Gaussian inference (VGI) for identification of limited, noise-distorted fluid rescsitation data

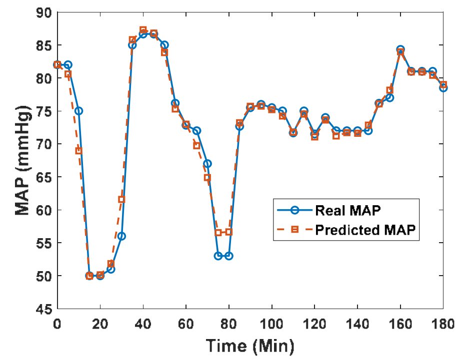

Figure 2. Measured and predicted MAP responses using RNSSM

TABLE I. Performance Metrics for All Animal Subjects

	RMSE (%)	MAE (%)	MDAPE (%)
MEAN	5.29	2.89	0.94
STD	3.10	1.55	0.40

IV. Conclusion

The RNSSM framework, integrating autoencoder learning and VGI, was developed for predicting hemodynamic responses in fluid resuscitation. Initial results implied the feasibility of the proposed approach for identifying reliable models from noisy, limited critical care data. Initial results encourage further investigation of the approach against existing digital twin models. Also, the robustness of RNSSM models in the presence of uncertainties should be further investigated in the near future.

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